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Bypassing Numerical Difficulties Associated with Updating Simultaneously Mass and Stiffness Matrices

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In a model updating problem, design parameters or entries of the finite element mass and stiffness matrices are tuned so that the adjusted structural dynamics model matches a set of identified modal parameters as closely as possible. Numerical difficulties are known to arise during this process because of the inverse nature of the problem. In this paper, we show that ill conditioning results from the disproportion between the mass derived and stiffness derived equations of the correction system. We discuss the effect of the resulting numerical difficulties on a class of sensitivity-based updating methods, and propose a two-step strategy to bypass them. In the first step, the correction system is nondimensionalized before it is solved in order to prevent large stiffness perturbations from masking mass errors. In the second step, the implementation of the singular value decomposition factorization is revisited to filter out all nonphysical contributions to the adjustment solution. The potential of this two-step strategy is demonstrated with the model refinement of a recently published planar frame benchmark problem that exhibits erroneous density parameters. Results obtained via the sensitivity-based element-by-element updating method are compared with those generated by a commercially available updating software.

Nomenclature

F_{ext}	= static loading matrix
H	= correction matrix
$\mathcal{I}$	= identity matrix
J	= minimization function of the updating problem
$K, k^{(e)}$	= analytical (FEM) stiffness matrix and elemental stiffness matrix
$L^{(e)}$	= localization operator to the eth finite element
$M, m^{(e)}$	= analytical (FEM) mass matrix and elemental mass matrix
p	= design parameters (physical parameters) of the model
R	= vector constructed from all residual forces
$\mathcal{R}$	= space of real numbers ($\mathcal{R} =]-\infty; +\infty[$)
$r_j^{(e)}$	= elemental vector of residual forces of the jth mode
U, V	= matrices of left and right singular vectors
λ_j	= jth ratio defined by Eq. (8)
ξ, ξ_C	= arbitrary quantity and arbitrary quantity characteristic of the problem
Σ, σ_j	= diagonal matrix of singular values and jth singular value
Φ	= identified mode shape matrix
Ψ	= measured static deflection matrix
Ω^2, ω_j^2	= identified eigenvalue matrix and jth identified eigenvalue
$()_1, ()_2$	= measured and nonmeasured components
$()_j$	= vector obtained from the jth column of a matrix quantity
$()_{\text{LFSVD}}, ()_{\text{RFSVD}}$	= quantities pertaining to the LFSVD and RFSVD algorithms
$\| \|_F$	= Frobenius (or Euclidean) norm ($\|A\|_F^2 = \sum_i \sum_j a_{i,j}^2$)
$(\bar{\ })$	= average quantity
$(\tilde{\ })$	= dimensionless scalar, vector, or matrix quantity
$()^+$	= pseudoinverse of a matrix quantity

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I. Introduction

HIGH-ACCURACY space structures require correlated finite element models for predicting their on-orbit dynamics whenever testing is not practical and for adjusting their control laws. During the test-analysis reconciliation step, numerical instability and ill conditioning occur because of the inverse nature of the updating problem where an adjusted finite element model (FEM) that matches a set of identified modal parameters is sought. A number of authors have already exposed the difficulties associated with solving this ill-conditioned updating problem without introducing unrealistic nonphysical corrections.[1–3] For example, it has been often observed that numerical instabilities tend to produce mass and stiffness corrections that are greater than 100% of the original values. To cope with this issue, Imregun et al.[2] have proposed a scaling procedure where all rows of the correction system are made to have the same largest entry. The singular value decomposition (SVD) has also been recommended for solving the correction system and filtering out its unstable singular vectors.[4,5] Moreover, the gap between the theory of SVD and the Lanczos and subspace iteration algorithms has been filled (see the work of Vogel and Wade,[6] for example), leading to efficient iterative SVD computational schemes which overcome most of the implementation challenges discussed by Ojalvo[3] and Ojalvo and Ting.[4] Recently, Avitabile and Li[1] have also shown that the outcome of an updating scheme depends on whether the mass and stiffness matrices are adjusted simultaneously or independently and that the computed solution is sensitive to the selection of the finite elements that are retained for adjustment.

Whether a sensitivity-based formulation,[7,8] an optimum matrix update scheme,[9,10] a perturbation algorithm,[11] or a pseudoeigenvalue assignment procedure[12] is selected for updating a given FEM, the numerical difficulties just described seem always to result from the disproportion between mass derived and stiffness derived equations of the correction system. Similar difficulties are encountered when transfer functions rather than modal parameters are used to update the model because of the high sensitivity of transfer functions to parameters such as the input and output locations and the frequency of interest.[13,14] These problems have not been addressed in depth by the model updating community. Rather, early strategies have preconized refining separately the mass and stiffness matrices.[9,15] The underlying motivation is the fact that it is usually easier to measure and model the mass than the stiffness or damping properties of a structural system. Once the mass matrix is accurately obtained the stiffness matrix is updated, and a better conditioning is usually achieved because the correction matrix is derived from one unique physical effect (stiffness) instead of a combination of two or possibly three distinct effects (inertia, stiffness, and damping). However, such an approach is not applicable to systems such as large orbiting structures that must be tested remotely, because in that case the mass, stiffness, and damping characteristics of the structure are unknowns that cannot be decoupled.

In this paper, we investigate the source of the numerical difficulties associated with the simultaneous updating of mass and stiffness properties and propose a two-step strategy to bypass them. First, we nondimensionalize the correction system before solving it in order to prevent large stiffness perturbations from masking mass errors. Next, we revisit the implementation of the SVD factorization to filter out all nonphysical contributions to the adjustment solution. The potential of this two-step strategy is demonstrated with the model updating of a recently published planar frame benchmark problem that exhibits erroneous density parameters. Results of the sensitivity-based element-by-element (SB-EBE) refinement are compared with those generated by a commercially available updating software.

II. Sensitivity-Based Element-by-Element Updating Procedure

Throughout this paper, we use the SB-EBE updating method[16,17] to address the various numerical difficulties associated with the updating problem. Therefore, we begin by overviewing the SB-EBE updating procedure in order to keep this paper self-contained.

The SB-EBE updating method is based on the minimization of the square of the Euclidean norm of the modal residuals computed from the identified modal parameters and the analytical mass and stiffness matrices

$$\min_{\{\delta p;\Phi_2;\Psi_2\}} J(\delta p;\,\Phi_2;\,\Psi_2) \tag{1}$$

where

$$J(\delta p;\,\Phi_2;\,\Psi_2) = \left\| \left(K(p+\delta p)\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} - \mathbf{M}(p+\delta p)\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix}\Omega^2 \right) \right\|_F^2 + \left\| \left(K(p+\delta p)\begin{bmatrix}\Psi_1\\ \Psi_2\end{bmatrix} - F_{\text{ext}} \right) \right\|_F^2 \tag{2}$$

The fundamental unknowns in Eq. (1) are the adjustments of the design parameters δp. Since the full instrumentation of a large structure is never possible, it is necessary to introduce also the unknowns Φ_2 and Ψ_2 which represent the nonmeasured components of the mode shapes and static deflections. In Ref. 16, a staggered iterative algorithm is described for solving the nonlinear coupled optimization problem (1). At each iteration n, the measured mode shape and static deflection vectors are first expanded to match the size of the FEM by solving the set of Euler's equations obtained for problem (1) with respect to Φ_2 and Ψ_2. Then, the design parameters are adjusted by an amount $\delta p^{(n)}$, where $\delta p^{(n)}$ is obtained from the solution of the second set of Euler's equations. In Ref. 16, it is shown that the equation $\partial J/\partial p = 0$ leads to the following rectangular overdetermined system:

$$H[p^{(n-1)}]\delta p^{(n)} = -R[p^{(n-1)}] \tag{3}$$

where

$$H[p^{(n-1)}] = \frac{\partial R}{\partial p}[p^{(n-1)}] \tag{4}$$

For dynamic mode shapes, R can be written as

$$R[p^{(n-1)}] = K[p^{(n-1)}]\begin{bmatrix}\Phi_1\\ \Phi_2^{(n)}\end{bmatrix} - M[p^{(n-1)}]\begin{bmatrix}\Phi_1\\ \Phi_2^{(n)}\end{bmatrix}\Omega^2 \tag{5}$$

Hence, the correction matrix H represents the sensitivity of the dynamic and static residuals R to adjustments of the FEM parameters. Using the assembly property of the FEM, the sensitivities are computed at the element level and algebraic formulas are available for expressing the partial derivatives of the elemental mass and stiffness matrices with respect to the parameters of the design. After $\delta p^{(n)}$ has been obtained by solving Eq. (3), the design parameters are adjusted with $p^{(n)} = p^{(n-1)} + \delta p^{(n)}$ and the mass and stiffness matrices are refined using the updated design parameters $p^{(n)}$. The efficiency of the SB-EBE updating scheme relies on its "zooming" capability. Essentially, only the elements connected to the degrees of freedom exhibiting the highest residuals are retained for updating, and only those parameters to which the residuals are highly sensitive are considered. After H is constructed, the correction system (3) is solved via an SVD factorization.

III. Singular Value Decomposition Algorithm

A. Overview of the SVD Algorithm

In general, the SVD algorithm is popular for solving overdetermined inverse problems because it provides an efficient computational scheme for Moore–Penrose's pseudoinverse.[18] The correction matrix is factored as

$$H = U\Sigma V^T \tag{6}$$

where U and V store the left and right singular vectors and verify $U^TU = \mathcal{I}$ and $V^TV = \mathcal{I}$, and Σ is a diagonal matrix that contains the singular values σ_j. In practice, the pseudoinverse H^+ of the rectangular system is never constructed, and the solution of Eq. (3) is obtained via a linear combination of the right singular vectors

$$\delta p = -\sum_{\{\sigma_j \geq \tau_{\text{LFSVD}}\}} \lambda_j V_j \tag{7}$$

where each scalar quantity λ_j is defined as the projection of the right-hand side on the jth left singular vector normalized by the corresponding singular value

$$\lambda_j = R^T U_j / \sigma_j \tag{8}$$

Usually, the set of singular vectors taken into account in Eq. (7) is designed to reject any singular value that is "small" compared to the others, which is representative of a singularity or ill conditioning. This is achieved by filtering out all singular vectors of the SVD factorization associated with singular values that are smaller than a threshold τ_{LFSVD} based on the mean of the distribution,

$$\tau_{\text{LFSVD}} = 10^{-q} \times \frac{1}{N}\sum_{j=1,\ldots,N} \sigma_j \tag{9}$$

where q is a user defined positive integer (typically, $5 \leq q \leq 10$) and N is the dimension of the unknown vector δp at a given iteration. In the sequel, this approach is referred to as the left filtered singular value decomposition (LFSVD) algorithm, because filtering is performed at the left of the system $H\delta p = -R$. This filtering strategy has been proposed by many authors, among which we cite Golub and Van Loan[18] and Maia,[5] and is widely accepted by the structural dynamics community. The eigensystem realization algorithm (ERA) developed by Juang and Pappa[19] for minimal-order state space system identification is an example as well as the MATLAB™ software.[20]

B. Discussion of the Conventional Filtering Strategy for Model Updating

The implementation of the SVD algorithm just discussed may produce nonphysical solutions when both mass and stiffness matrices are refined simultaneously. These nonphysical solutions are characterized by very large adjustments—typically more than +/−100% variation of the original values—which must be eliminated if any physically sound refined model is to be obtained. The mechanism by which large entries corrupt the adjustment vector is revealed in Eq. (7). Since the right singular vectors V_j are normalized to satisfy the orthogonality condition, their entries are necessarily in the range $[-1; +1]$. Therefore, large correction parameters have to come from large ratios $|\lambda_j|$ in Eq. (7). Of course, most of them are caused by small singular values given that

$$\lim_{\sigma_j \to 0} |\lambda_j| = +\infty \tag{10}$$

However, our experience with various studies using both numerically simulated and experimental data indicates that some $|\lambda_j|$ quantities can be large even for $\sigma_j \geq \tau_{\text{LFSVD}}$. The existence of such large ratios can be attributed to the combination of the localized nature of the error in the updating problem, and to the ill conditioning of the correction matrix. Generally, ill conditioning results from introducing in the system of Eq. (3) sensitivity terms with respect to mass and stiffness design parameters and sizing variables which have dissimilar orders of magnitude.

Next, we propose a right filtered singular value decomposition (RFSVD) algorithm for resolving these numerical problems by filtering out the large ratios $|\lambda_j|$. However, the reader should be aware that we are not implying that these large ratios must always be eliminated. In fact, they should be kept in the solution when large but meaningful modifications of the FEM matrices are sought. Nevertheless, when it is known that the adjustment vector must satisfy a constraint dictated by physical considerations (for example, a thickness parameter should not be decreased to the point where it becomes negative), it makes perfect engineering sense to filter out these ratios whether they are associated to "large" singular values or "small" ones. Moreover, most updating methods solve iteratively a linear system which is derived via a first-order Taylor's expansion formula. The underlying assumption is that the change brought to the model should be small enough to neglect the higher order terms of the expansion. Even though large structural modifications may be brought to the FEM by cumulating a number of small perturbations, the assumption $\|\delta p\|_F$ is small should be satisfied at each iteration, which further justifies the need for keeping the adjustment vector δp uncorrupted from large ratios $|\lambda_j|$.

IV. Strategies for Bypassing Numerical Difficulties

A. Right Filtered Singular Value Decomposition Algorithm

Modifying the conventional left filtering strategy consists essentially in replacing in Eq. (7) the set $\{\sigma_j \geq \tau_{\text{LFSVD}}\}$ by the modified set $\{|\lambda_j| \leq \tau_{\text{RFSVD}}\}$ where

$$\tau_{\text{RFSVD}} = \epsilon \times \frac{1}{N} \sum_{j=1,\ldots,N} |\lambda_j| \tag{11}$$

and ϵ is a user-defined positive floating number in the range $[0; 1]$. The difference between LFSVD and RFSVD is not restricted to the fact that the latter algorithm filters out the large nonphysical adjustments. It also adds to the solution (7) contributions of the factorization that are associated with small singular values assuming that the corresponding ratios $|\lambda_j|$ are smaller than τ_{RFSVD}. Therefore, the adjustment δp_{LFSVD} that would be obtained with the LFSVD implementation is modified by 1) removing the components that tend to produce large nonphysical adjustments and 2) adding components in the "numerical null space" of the correction matrix H^T,

$$\delta p_{\text{RFSVD}} = \delta p_{\text{LFSVD}} - \sum_{\{j:1\leq j\leq N;|\lambda_j|>\tau_{\text{RFSVD}}\}} \lambda_j V_j + \sum_{\{j;1\leq j\leq N;\sigma_j<\tau_{\text{LFSVD}} \text{ and } |\lambda_j|\leq\tau_{\text{RFSVD}}\}} \lambda_j V_j \tag{12}$$

The term numerical null space is used here by opposition to mathematical null space to denote the space spanned by the right singular vectors corresponding to singular values that are smaller than the cutoff τ_{LFSVD}: $\sigma_j < \tau_{\text{LFSVD}} \Leftrightarrow \mathbf{H}\mathbf{V}_j \approx \mathbf{0}$. By contrast, the mathematical null space would be spanned by the set of right vectors with singular values exactly equal to zero. Since we are mostly concerned with practical numerical issues, the term null space always refers to the numerical null space in the remainder of this paper. It is important to note that contributions in the null space of matrix H^T are added only if the corresponding ratios $|\lambda_j|$ are small enough in Eq. (12). Hence, RFSVD adds contributions from the null space that are consistent (orthogonal) with the right-hand-side vector. These consistent contributions add important information that would be filtered out otherwise.

B. Investigation of the Mechanism of Ill Conditioning for the Sensitivity-Based Element-by-Element Updating Method

So far, we have attributed ill conditioning to the disproportion between mass derived and stiffness derived equations, making the aforementioned modified filtering strategy applicable to most updating techniques. Here, the mechanism of ill conditioning is investigated in details for the particular case of the SB-EBE updating method. However, the conclusions presented herein pertain to all model updating strategies that are based on the concept of residual forces[21,22] whether these residuals are formulated at the elemental level or not.

$$r_j^{(e)} = \left(k^{(e)} - \omega_j^2 m^{(e)}\right) L^{(e)} \begin{bmatrix} \Phi_{j1} \\ \Phi_{j2} \end{bmatrix} \tag{13}$$

The mechanism of ill conditioning is revealed by identifying the dominant contributions from geometrical and material properties in Eq. (13). For example, the planar Euler–Bernouilli beam element used in the numerical example to follow involves essentially axial and bending effects, and the corresponding dominant factors in the elemental mass and stiffness matrices are

$$m^{(e)} \approx \begin{cases} \text{axial} & \rho A L \\ \text{bending} & \dfrac{\rho A L^3}{210} \end{cases} \qquad k^{(e)} \approx \begin{cases} \text{axial} & \dfrac{EA}{L} \\ \text{bending} & \dfrac{4EI}{L} \end{cases} \tag{14}$$

Substituting these factors in Eq. (13) provides an estimate of the residual's magnitude for both axial and bending effects,

$$r_j^{(e)} \approx \begin{cases} \text{axial} & A\left(E - \omega_j^2 \rho L^2\right) \\ \text{bending} & \left(\dfrac{4EI}{L} - \omega_j^2 \dfrac{\rho A L^3}{210}\right) \end{cases} \tag{15}$$

Hence, for realistic geometrical and material properties, the stiffness contribution to the distribution of the elemental residuals $r_j^{(e)}$ always dominates the contribution of the mass by several orders of magnitude since $(EA) \gg (\omega_j^2 \rho A L^2)$ and $(4EI/L) \gg [\omega_j^2(\rho A L^3/210)]$. The conclusion is straightforward: modeling errors producing discrepancies in the stiffness matrix are always more visible than modeling errors affecting the mass matrix. This explains why residual-based and sensitivity-based algorithms, including the SB-EBE updating method, tend to adjust systematically the stiffness parameters even in cases where the modeling error affects the mass matrix only.

C. Defining Dimensionless Systems

In Eq. (13), the mass contribution could be increased by updating the model with high-frequency modes. In practice, higher order modes are difficult to identify accurately and, therefore, this solution is unfeasible for most applications. An alternative solution is presented here; mass and stiffness contributions are balanced by introducing dimensionless geometrical and material properties and carrying them throughout the updating procedure to obtain

$$\tilde{E}\tilde{A} \approx \tilde{\omega}_j^2 \tilde{\rho} \tilde{A} \tilde{L}^2 \quad \text{and} \quad \frac{4\tilde{E}\tilde{I}}{\tilde{L}} \approx \tilde{\omega}_j^2 \frac{\tilde{\rho}\tilde{A}\tilde{L}^3}{210} \tag{16}$$

Basically, all geometrical and material properties are made dimensionless by dividing them by a characteristic quantity of same unit ξ_C,

$$\tilde{\xi} = \xi / \xi_C \tag{17}$$

where ξ may represent a length, area, thickness, Young's modulus, density, etc. In addition to the structure's geometrical and material

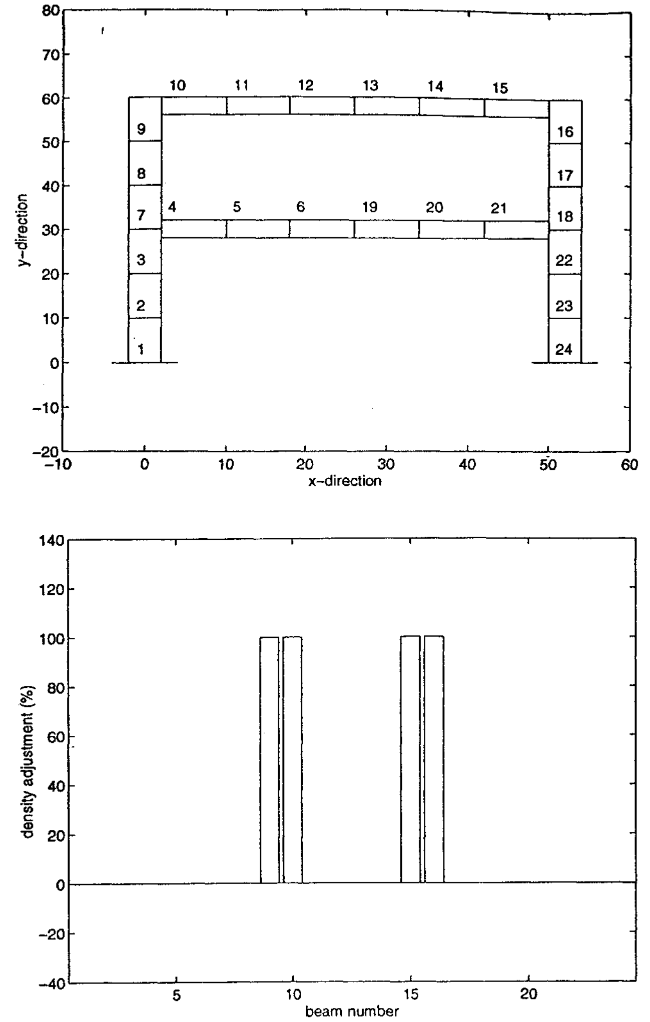

Fig. 1 **Planar frame structure with density perturbation at finite elements 9, 10, 15, and 16.**

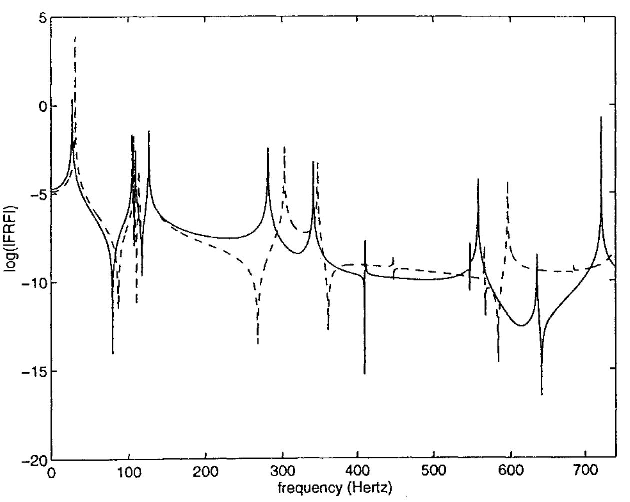

Fig. 2 **Comparison of typical undamped frequency response functions before updating the model; — test data, - - analytical model.**

properties, the forces, moments, frequencies, and mode shapes are also normalized. Usually, the characteristic quantities ξ_C are obtained by averaging the parameters ξ of the model. More complex formulas may also be defined for enforcing consistency between various dynamical effects. Table 1 provides some typical normalization factors used in the numerical example presented in Sec. V. The model is updated using the dimensionless quantities rather than the original dimensions, and the updating results are transformed

Table 1 Typical normalization factors for the dimensionless updating formulation

Parameter	Symbol	Dimensionless factor
Length	L	$L_C = \bar{L}$
Area	A	$A_C = \bar{A}$
Young's modulus	E	$E_C = \bar{E}$
Density	ρ	$\rho_C = \bar{\rho}$
Poisson's ratio	ν	$\nu_C = 1.0$
Moment of Inertia	I	$I_C = \bar{A}^2$
Thickness	h	$h_C = \bar{L}$
Displacement	U	$U_C = \bar{L}$
Rotation	θ	$\theta_C = 1.0$
Force	F	$F_C = \bar{E} \times \bar{A}$
Moment	$\mathcal{M}$	$\mathcal{M}_C = \bar{E} \times \bar{A}^2/\bar{L}$
Eigenvalue	ω_j^2	$\omega_C^2 = \bar{E}/\bar{\rho} \times \bar{A}$

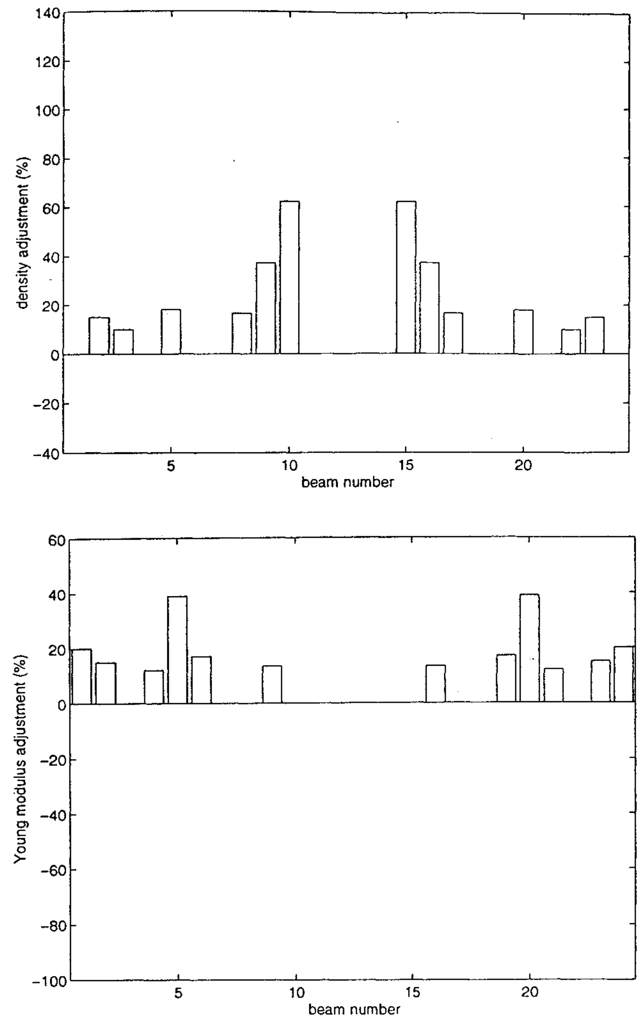

Fig. 3 **Density and Young's modulus adjustments for updating case 1, SYSTUNE, modes 1-11.**

back into the physical space by multiplying them by the characteristic factors ξ_C.

V. Numerical Example

A. Description of the Test Structure

The ideas exposed in this paper are validated with the FEM updating of the planar frame structure investigated by Avitabile and Li[1] (Fig. 1). This focus structure has aluminum tubular cross sections. It is attached to the ground with springs that simulate elastic boundary conditions. The corresponding FEM contains 24 beam elements and generates 72 degrees of freedom (DOF). The modal test is simulated by performing the eigenanalysis of a perturbed

Table 2 Test-analysis correlation before updating

Mode	$\omega_{test}/2\pi$, Test frequency, Hz	Error in frequency,[a] %	Diagonal MAC value,[b] %
1	27.8	13.37	99.95
2	105.1	1.40	99.31
3	109.7	1.07	99.47
4	126.7	3.23	99.82
5	281.7	0.02	99.99
6	341.7	3.83	99.15
7	410.6	10.50	98.95
8	547.4	2.11	97.43
9	558.0	9.10	98.62
10	634.9	7.99	98.51

[a]$\text{Error}_j = 100 \times |\omega_{FEM,j} - \omega_{test,j}|/\omega_{test,j}$.
[b]$\text{MAC}_{i,j} = 100 \times (\Phi_{FEM,i}^T \Phi_{test,j})^2/[(\Phi_{FEM,i}^T \Phi_{FEM,i})(\Phi_{test,j}^T \Phi_{test,j})]$.

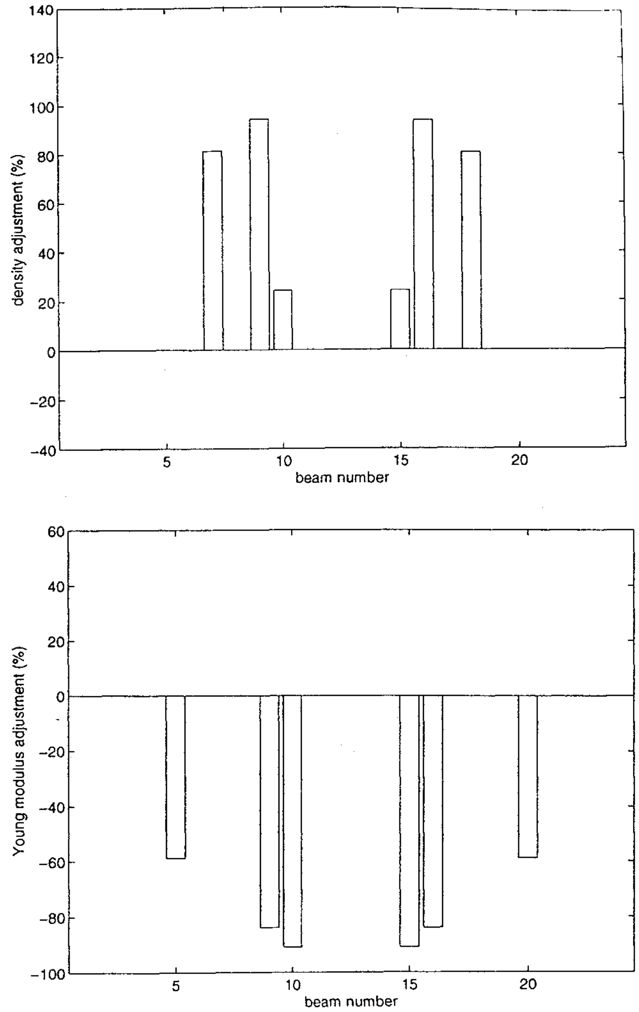

Fig. 4 Density and Young's modulus adjustments for updating case 2, SB-EBE updating, modes 1–11, LFSVD, physical model.

model. Unlike in Ref. 1, only the translational DOFs are assumed to be measured. Therefore, modal expansion is required to expand the "experimental" mode shape vectors from 48 to 72 DOFs during the updating procedure. This deviation with respect to the problem discussed by Avitabile and Li is introduced to represent more realistic (and difficult) conditions for an updating algorithm. Following Ref. 1, the perturbed model is derived from the analytical (unperturbed) model by increasing the density parameter of the four upper corner elements by 100% (see Fig. 1). It can be observed from Table 2 that the test-analysis correlation before updating is acceptable even though the four corner elements are erroneously modeled. However, Fig. 2 shows significant variations in the frequency response functions (FRF) of the undamped model.

B. Verification of the Claim on the Source of Ill Conditioning

Figure 3 displays the adjustment obtained with the commercial software SYSTUNE™[23] when its sensitivity-based updating procedure[24] is fed with the first 11 modes. The reader can observe the algorithm's clear tendency to adjust the beam elements that are close to the frame's cantilever boundaries where most of the strain energy is concentrated. From the significant adjustment of the Young's modulus parameters it follows that the updating algorithm cannot distinguish between mass related and stiffness related errors.

On the other hand, the SB-EBE updating algorithm is shown to produce better results than SYSTUNE for the same input modal set. Figure 4 shows that the erroneous areas are correctly located by the SB-EBE updating algorithm and that the influence of the cantilever boundary conditions on the adjustment is minimized. However, as in the case of SYSTUNE, the SB-EBE updating procedure cannot distinguish between mass and stiffness errors. Figure 5 indicates that the large ratios $|\lambda_j|$ are not systematically caused by small singular values. Nevertheless, 20 out of 30 of them are greater than 10^{+5} and result into more than 50% perturbation of the original Young's moduli, which supports our claim. Half of them originate from large sensitivities with respect to density parameters in columns 1–15, which also supports our claim.

Next, adjustment is performed with the SB-EBE updating algorithm and 20 modes instead of 11. Modes 12–20 exhibit higher bending deformations that are localized to the two horizontal longerons. Therefore, these modes store more strain energy in the vicinity of the areas with modeling errors, which should improve the updating results (see, for example, Ref. 25). This is clearly demonstrated in

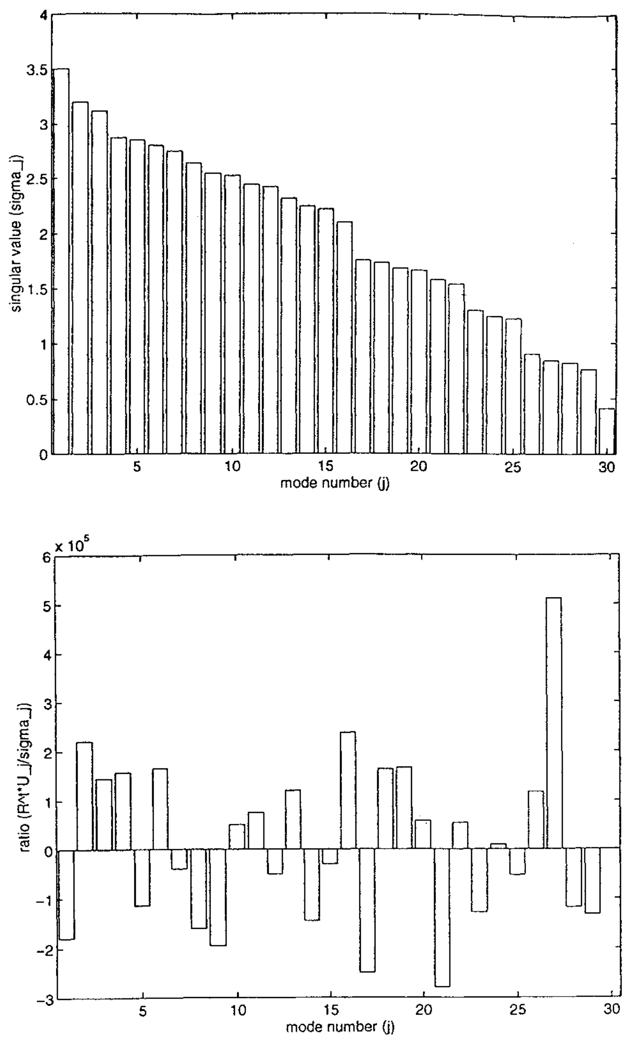

Fig. 5 Distribution of singular values and ratios λ_j for updating case 2, densities correspond to columns 1–15 and Young's moduli correspond to columns 16–30.

Table 3 Average mass and stiffness adjustments for various updating configurations

Definition	Updating Case	$\bar{E}$,[a] %	$\bar{E}$,[b] %	$\bar{\rho}$,[a] %	$\bar{\rho}$,[b] %
SYSTUNE, 11 modes physical model	1	+6.0	+9.9	+50.0	+5.5
SB-EBE updating, 11 modes physical model, LFSVD	2	−85.0	−6.0	+60.0	+8.0
SB-EBE updating, 20 modes physical model, LFSVD	3	−34.8	−6.5	+45.3	+2.3
SB-EBE updating, 11 modes physical model, RFSVD	4	−2.6	+0.5	+48.9	+8.3
SB-EBE updating, 20 modes physical model, RFSVD	5	0.0	−0.1	+110.8	+8.5
SB-EBE updating, 11 modes dimensionless model, RFSVD	6	0.0	−0.4	+60.0	+15.4
SB-EBE updating, 20 modes dimensionless model, RFSVD	7	0.0	0.0	+105.0	0.0
Perturbation		0.0	0.0	+100.0	0.0

[a] average adjustment for the mismodeled elements 9, 10, 15 and 16.
[b] average adjustment for the 20 correct elements.

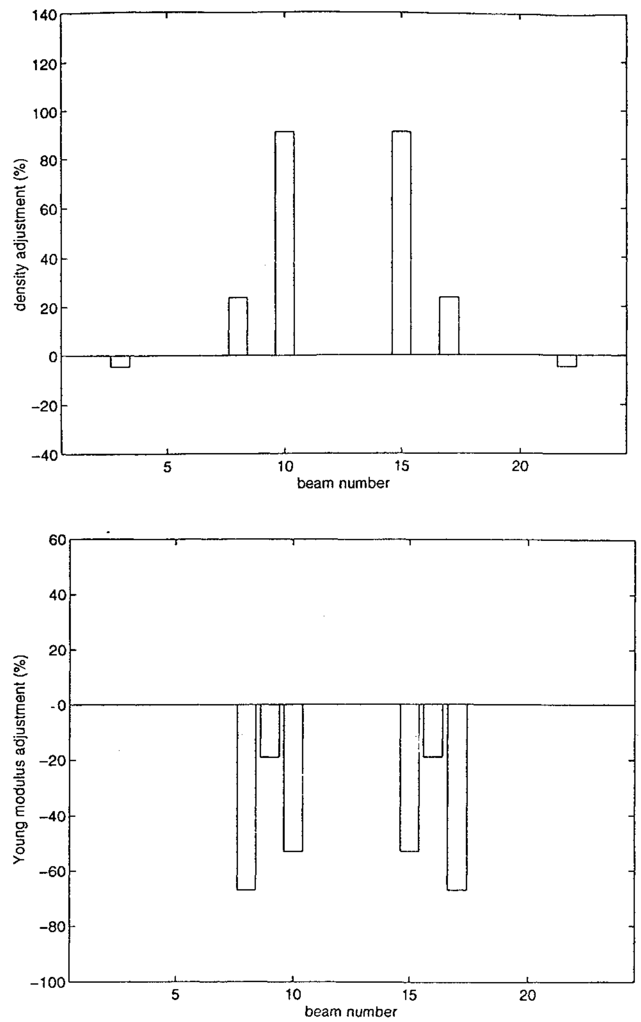

Fig. 6 Density and Young's modulus adjustments for updating case 3, SB-EBE updating, modes 1–20, LFSVD, physical model.

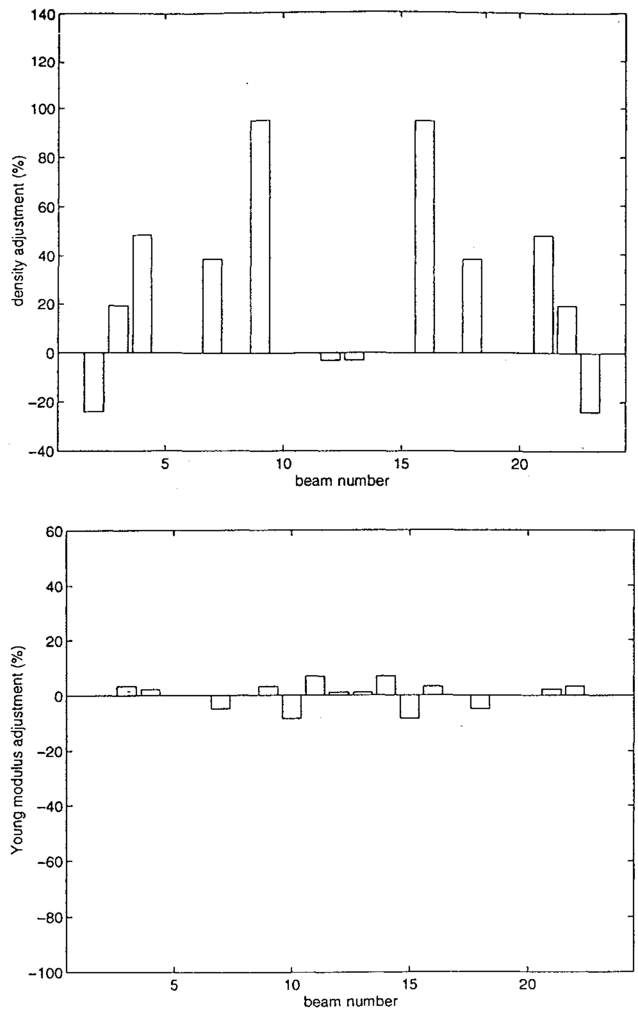

Fig. 7 Density and Young's modulus adjustments for updating case 4, SB-EBE updating, modes 1–11, RFSVD, physical model.

Fig. 6 where the amount of extraneous adjustment is reduced by more than half. However, these results also show that enriching the input modal set is not sufficient for decoupling the mass and stiffness errors.

C. Improvement of the Model Updating Procedure

Table 3 summarizes the results obtained with both updating algorithms. It lists the average mass and stiffness adjustments witnessed by the four erroneous beams and the 20 correct elements. Ideally, these should be $\bar{\rho} = +100\%$ and $\bar{E} = 0\%$ for the set of erroneous elements, and $\bar{\rho} = \bar{E} = 0\%$ for the correct ones. The advantages of filtering large ratios $|\lambda_j|$ rather than small singular values σ_j is clearly demonstrated in Table 3 and Figs. 7 and 8. Eliminating these large ratios improves the identification of the source of error because of the combined effects of 1) removing the large modifications of stiffness properties that were affecting previous updating results (cases 1–3) and 2) providing more accurate corrections of

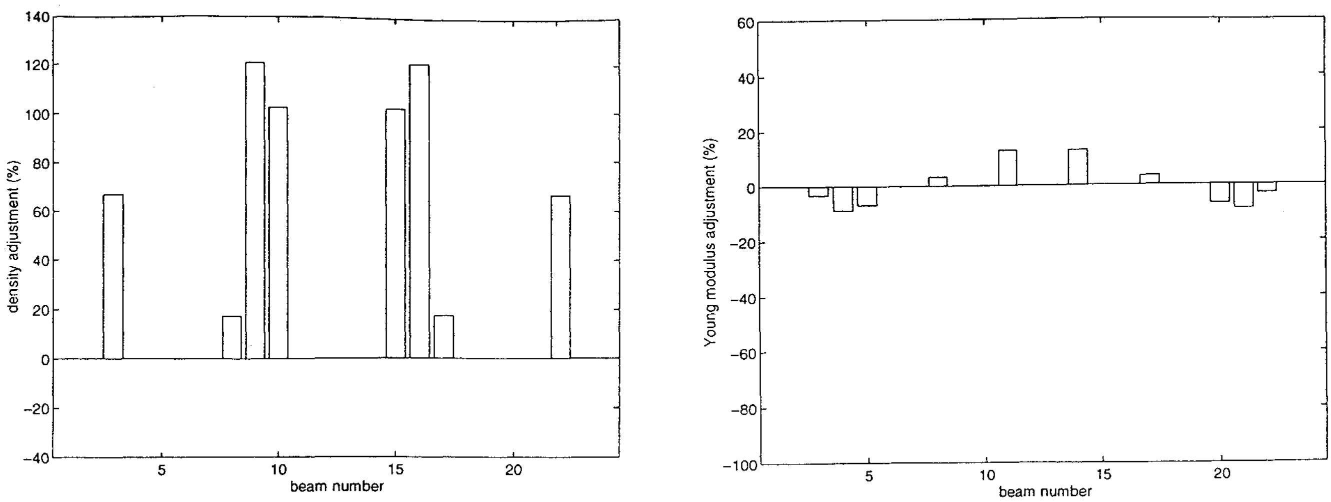

Fig. 8 Density and Young's modulus adjustments for updating case 5, SB-EBE updating, modes 1–20, RFSVD, physical model.

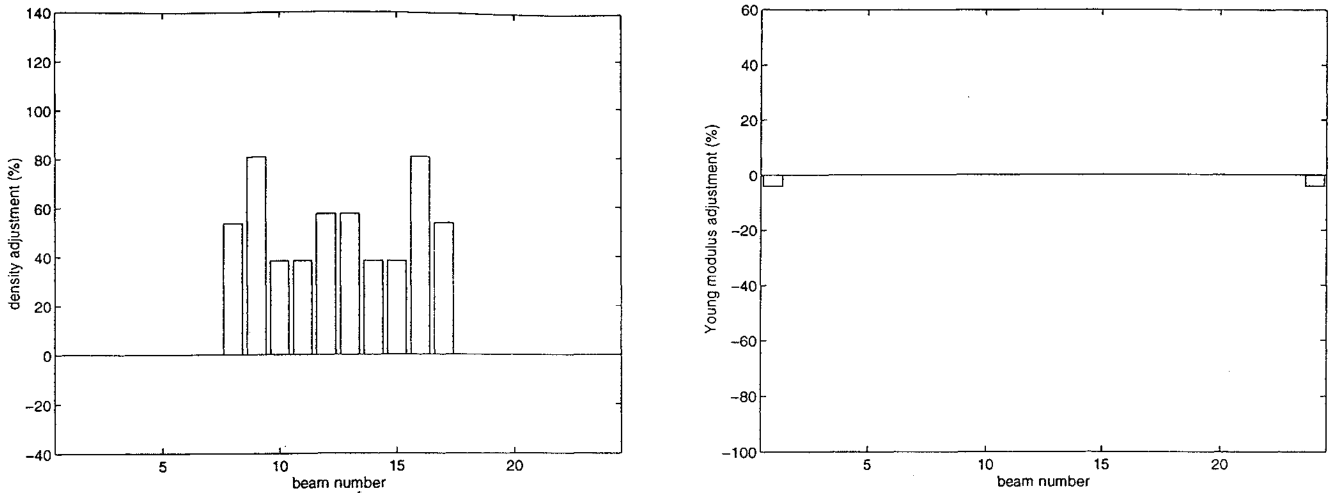

Fig. 9 Density and Young's modulus adjustments for updating case 6, SB-EBE updating, modes 1–11, RFSVD, dimensionless model.

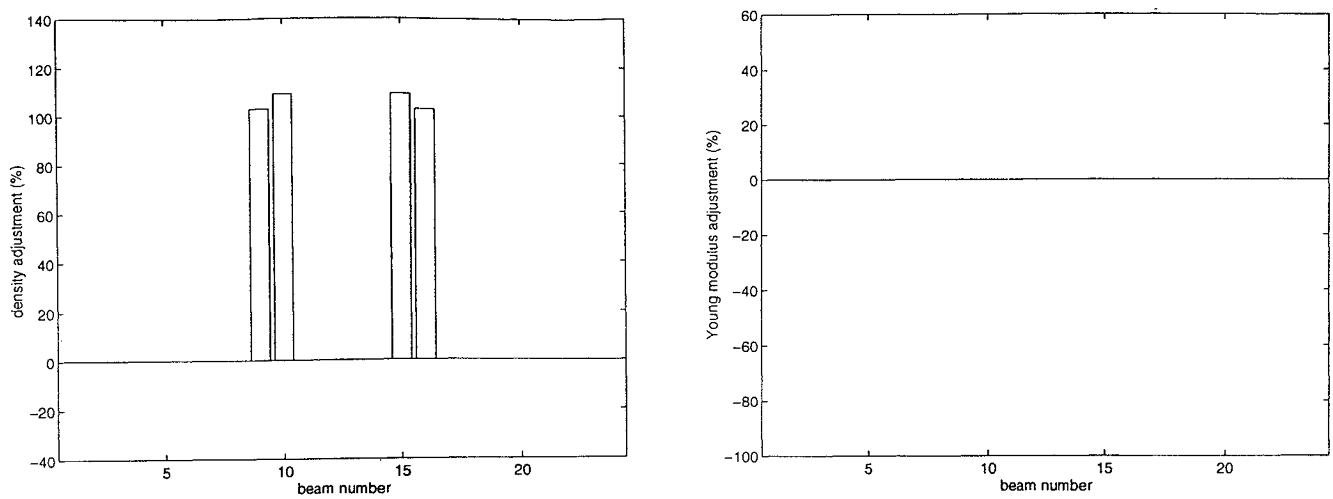

Fig. 10 Density and Young's modulus adjustments for updating case 7, SB-EBE updating, modes 1–20, RFSVD, dimensionless model.

the density parameters. Figure 7 shows that the density adjustment based on 11 modes tends to be distributed across the entire structure because the updating algorithm is not provided with enough local information that would enable it to distinguish between many solutions that are equally acceptable in a least-squares sense. Enriching the modal set with higher order local bending effects improves significantly the adjustment of the mass matrix as shown in Fig. 8. The modeling error is correctly located at the four upper corner elements (except for two beams located near the cantilever roots of the structure), and the source of error is correctly identified as the density parameter. As a result, the average error for the first 10 updated frequencies is equal to 1.84% +/− 0.49% only, and the corresponding modal assurance criterion parameters are greater than 99.5%.

Cases 6 and 7 correspond to the refinement of a dimensionless FEM with the SB-EBE updating method, the modified filtering strategy RFSVD, and 11 and 20 input modes. Figure 9 shows that the dimensionless updating based on 11 modes locates the modeling error at the upper longeron and identifies its source as the density. The exact solution is obtained in case 7 when 20 modes are inputed as illustrated in Fig. 10. This example highlights the virtues of the dimensionless formulation which prevents the mass and stiffness errors from masking each others.

VI. Conclusion

An investigation of the numerical problems that arise during the updating of finite element models is presented. It is found that ill conditioning may be attributed to the disproportion between mass derived and stiffness derived equations that are mixed together when their corresponding matrices are updated simultaneously. It is also demonstrated that conventional SVD solvers that filter the smallest singular values of the correction matrix do not always resolve these numerical difficulties. An alternative strategy is proposed, where the largest contributions of the right-hand-side vector to the information content of the correction system are filtered. This filtering strategy eliminates parasitic nonphysical adjustments and, in some cases, adds to the updating solution important components that are erroneously filtered out by a conventional SVD solver. It is also shown that nondimensionalizing a finite element model before updating it prevents stiffness perturbations from masking mass errors. The benefits of the combination of the proposed modified SVD solver and the nondimensionalization procedure are demonstrated with the updating of a recently published planar frame benchmark problem. The results obtained via the sensitivity-based element-by-element refinement[17] method are also much superior to those generated by a commercially available updating software.

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